Mathematical models
in
environmental, crop and food sciences

STAAA

Doctoral School in Agricultural, Environmental and Food Sciences

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Models & Modelling

- Technology
- Machines & Games
- System Theory

PART II - Model Languages

- **Model**
- **Math Formalism**

Simulation

- Programming
- Case Studies with Matlab

MODEL .. as a software ?

- **LAND management (GIS platform plugins)**
  - **Land Use**
  - **Basin / catchment (Runoff-Erosion)**

- **FARM DSS (Decision Support Systems)**
  - **Economical simulations**
  - **Irrigation** (e.g.Cropwat / Aquacrop)

- **Cropping System**
  - **Crop Yield Simulator (DSSAT, Stics, Apsim)**
  - **Soil Biochemistry (OM, Nitrogen, root)**

- **Community & population dynamics**
  - **Response to environment**
  - **Interactions (agents)**

APP ?

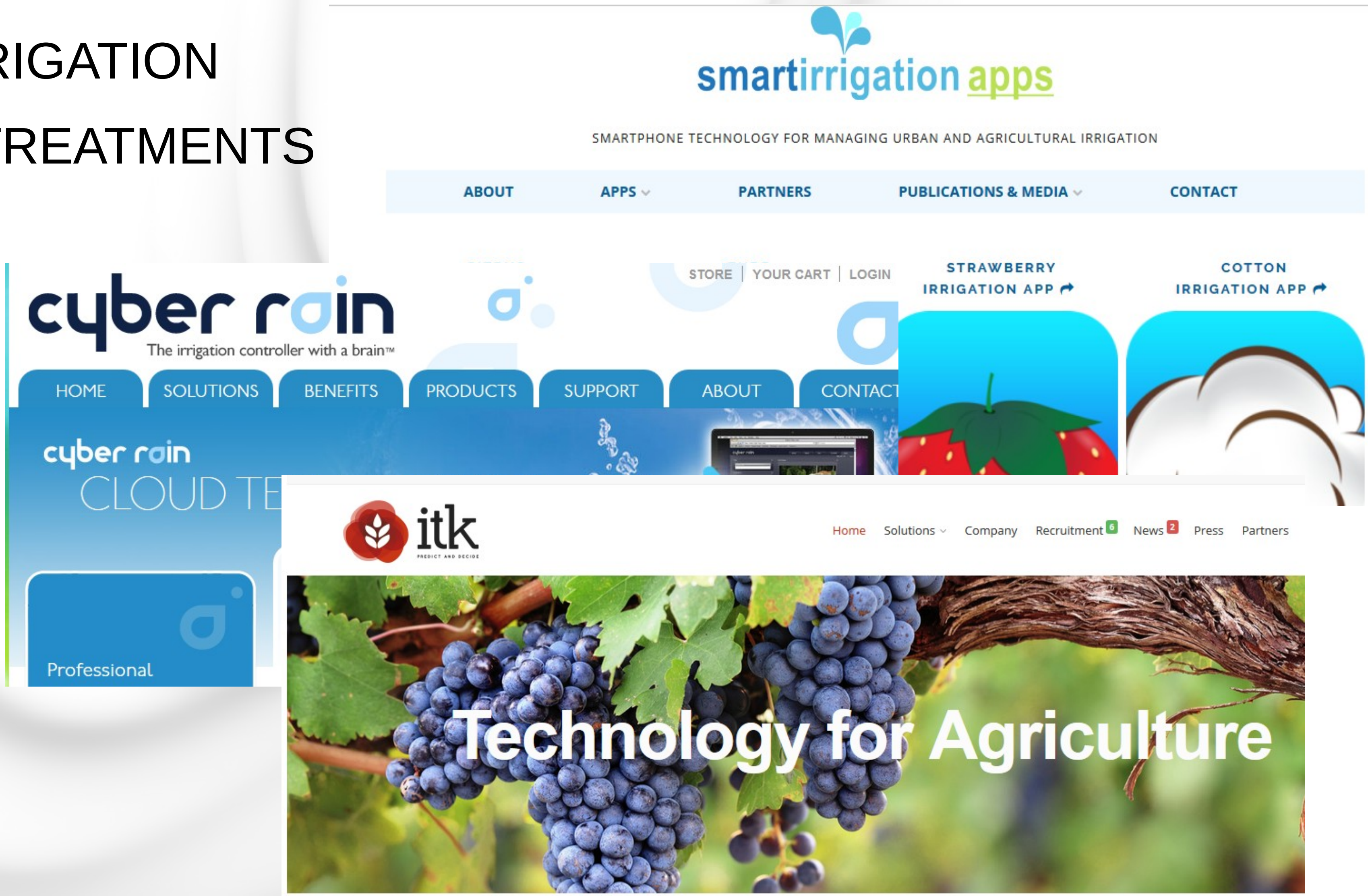
Definition: APP is an abbreviation for application. An app is a piece of software. It can run on the Internet, on your computer, or on your phone or an electronic device. The word "app" is a more modern usage, but this is really the same thing as a program.

http://google.about.com/od/a/g/apps_def.htm

APPs and Agriculture

IRRIGATION

& TREATMENTS



..so what is a MODEL ?

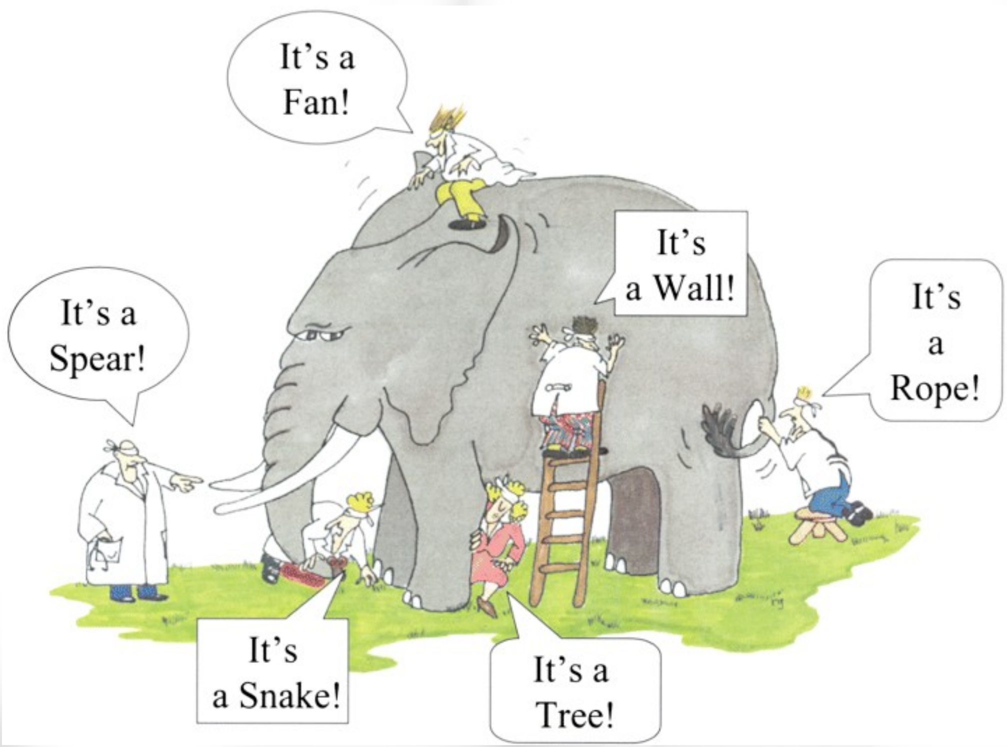
a MODEL is a
simplified representation
of a
REAL SYSTEM

..better say a

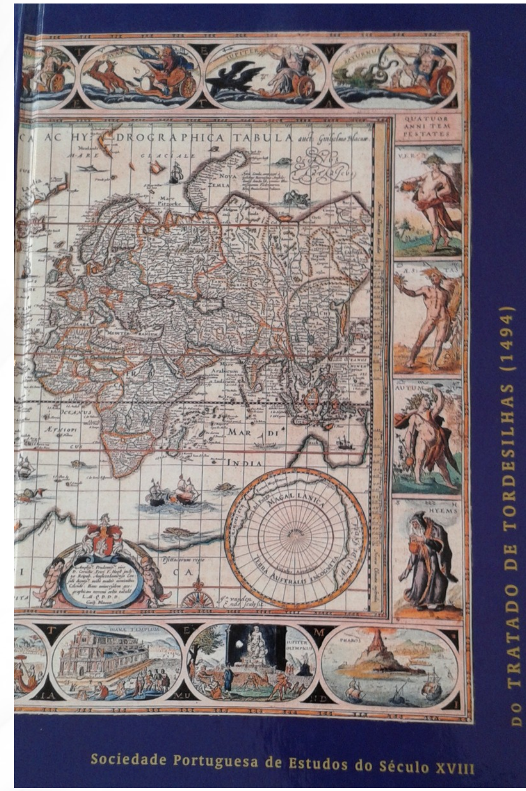
a system's MODEL

Modeling an EXISTING system

a tale about perception and observation (and hypothesys)



the tale of the blind man



the treatise of Tordesilhas stated a divide along a meridian 370 leagues west of the Cape Verde islands

https://en.wikipedia.org/wiki/Blind_men_and_an_elephant

MODEL types

- LANGUAGE LEVEL and COMPLEXITY
  - Conceptual M., Simulation M., Mathematical M., ..
- EXTERNAL MODEL .. analyze /sets INPUT
  - Environment (e.g. grow chamber)
  - User's behavior, Control actions
- INTERNAL MODEL
  - SYSTEM analysi focuses on STATE VARS

Communicate Structures & Relations
PURPOSES

Modeling structures

Modeling organisation

Modeling dynamics

with the aim of

share vision

Build an Operative model

understand and discuss results

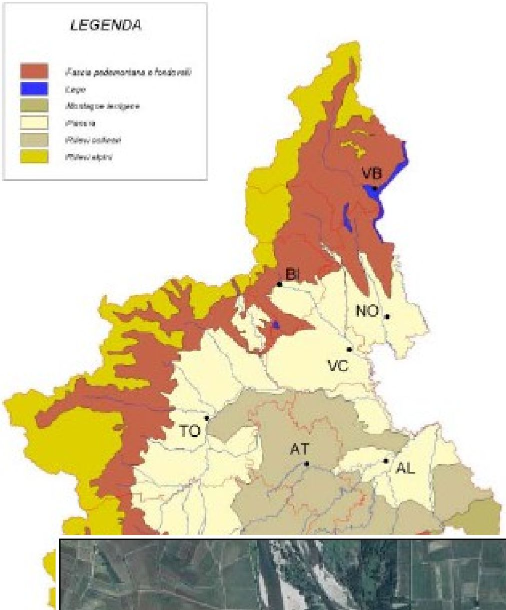
Communicate Structures & Relations

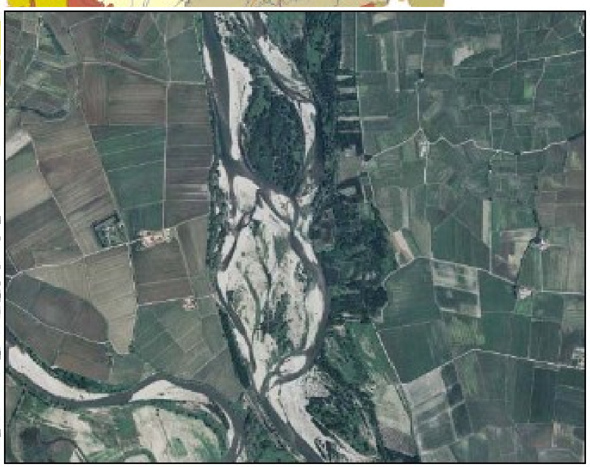
Symbol dictionary (grammar)
Syntax (rules)

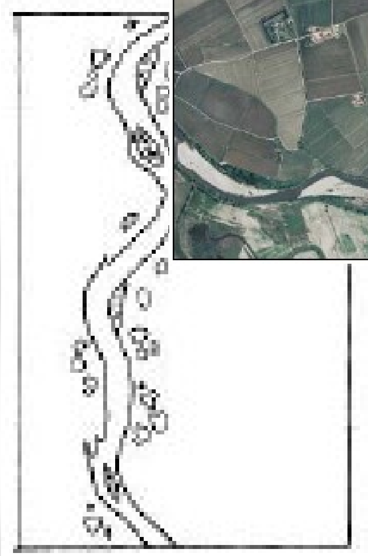
LANGUAGE TYPES
Graphic (universal)
Formal
- alphabet + sintactical rules
- algebraic rules (calculus)

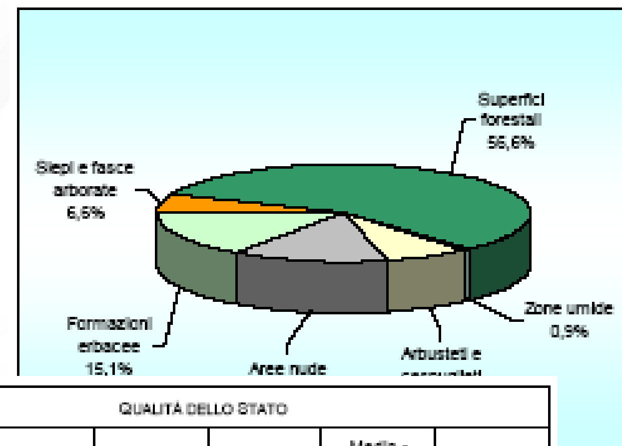
Translators / Converters / INTERFACES

Communicating Structures



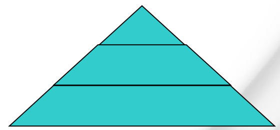






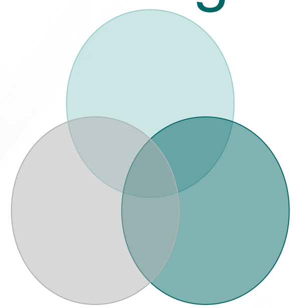
	QUALITÀ DELLO STATO					
		Alto	Medio - alto	Medio	Medio - basso	Basso
ENTITÀ DELLE PRESSIONI	Alto	7	8	8	9	10
	Medio – alto	6	7	8	8	9
	Medio	5	6	7	7	8
	Medio – basso	3	4	5	6	7
	Basso	1	2	3	4	5

Organization

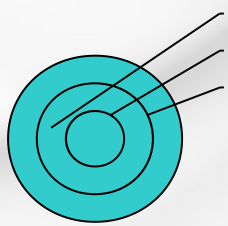


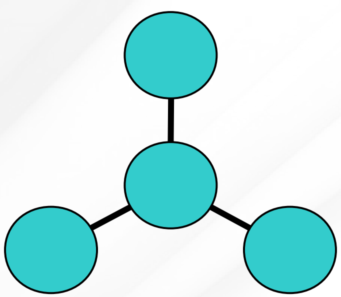
Pyramidal Diagram

Venn Diagram



Target Diagram

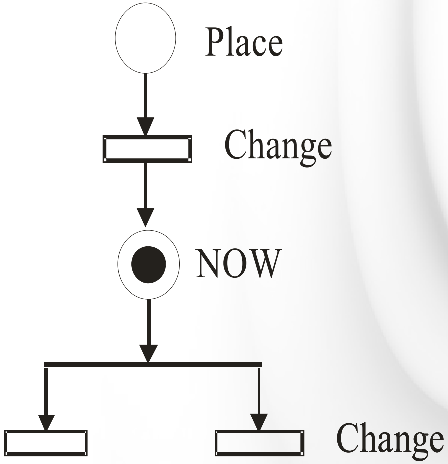




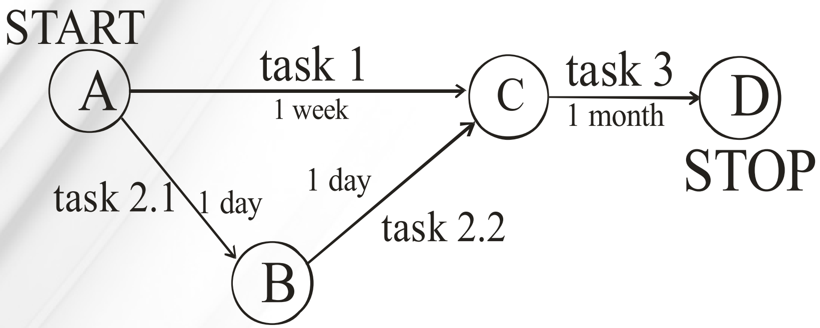
Radial Diagram

DISCRETE TIME STEP

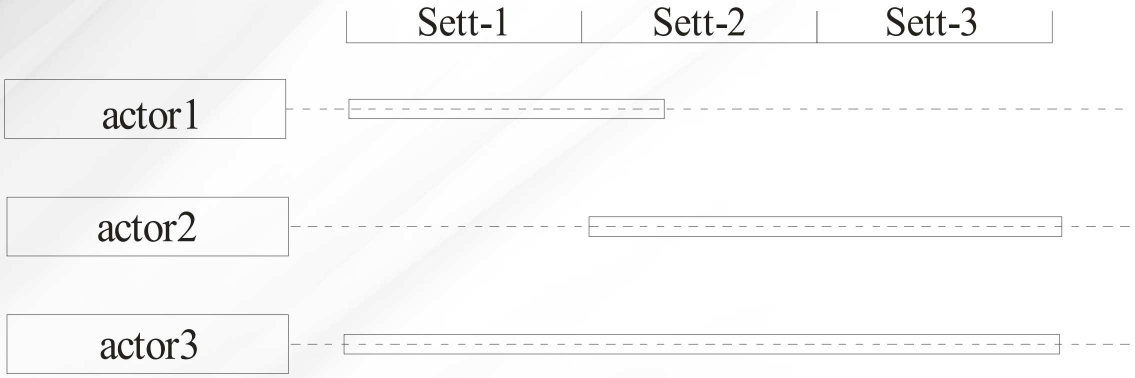
PETRI net



Pert chart / diagram

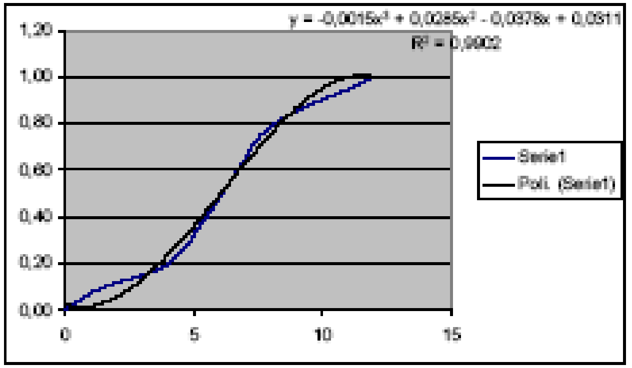


Gantt diagram / chart

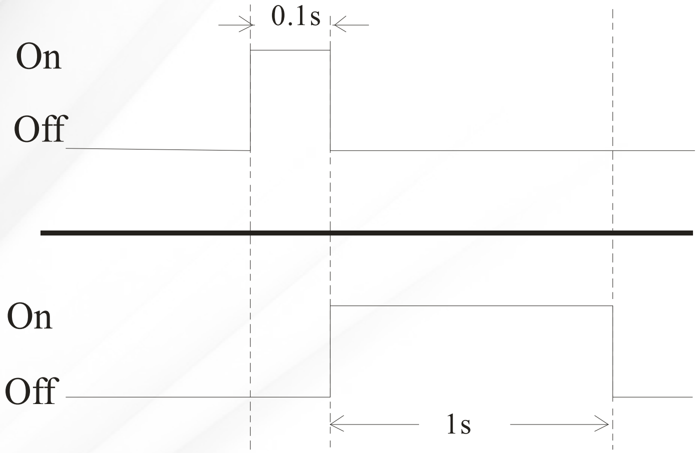


Continuous Dynamics

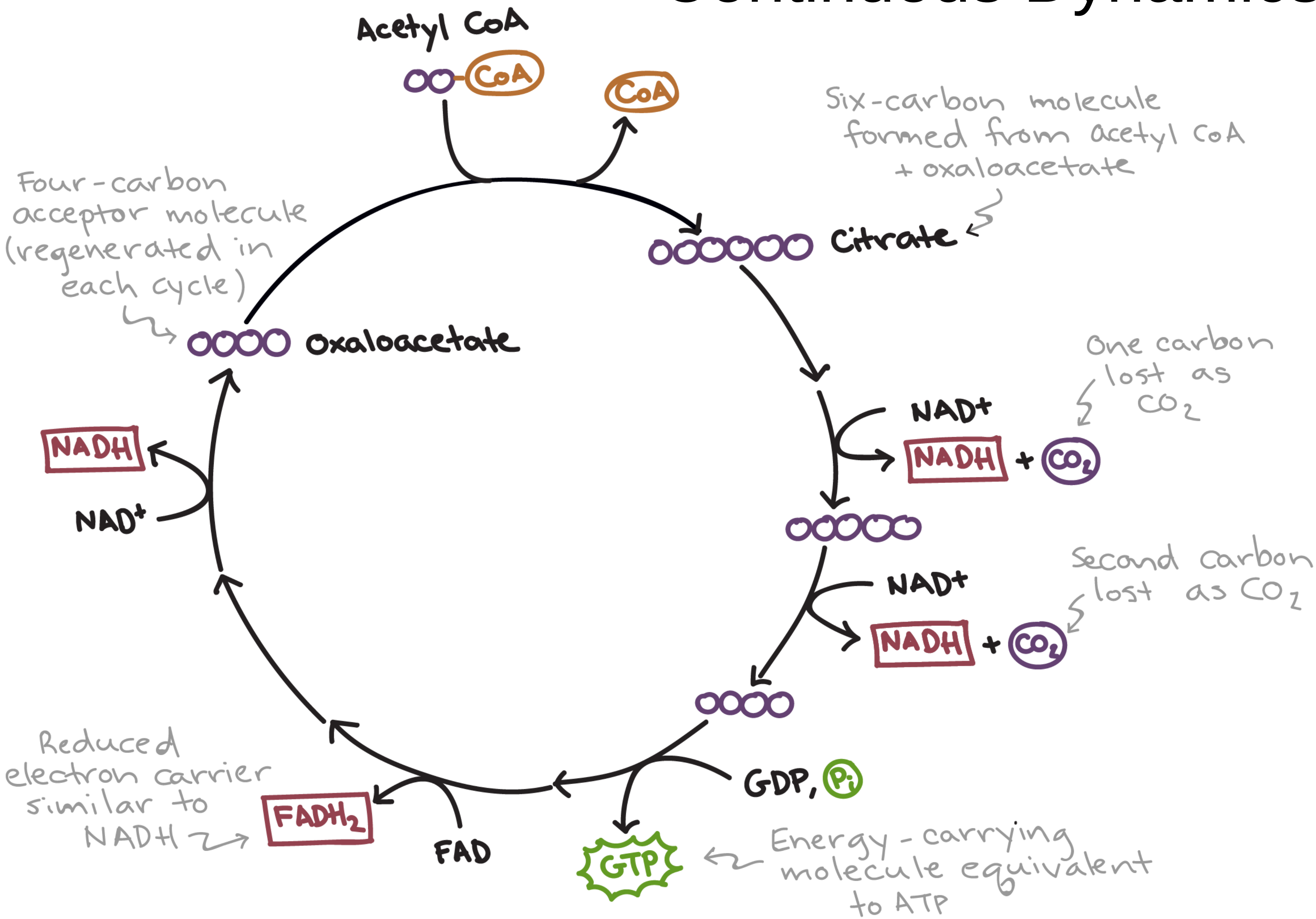
Plots / Trends



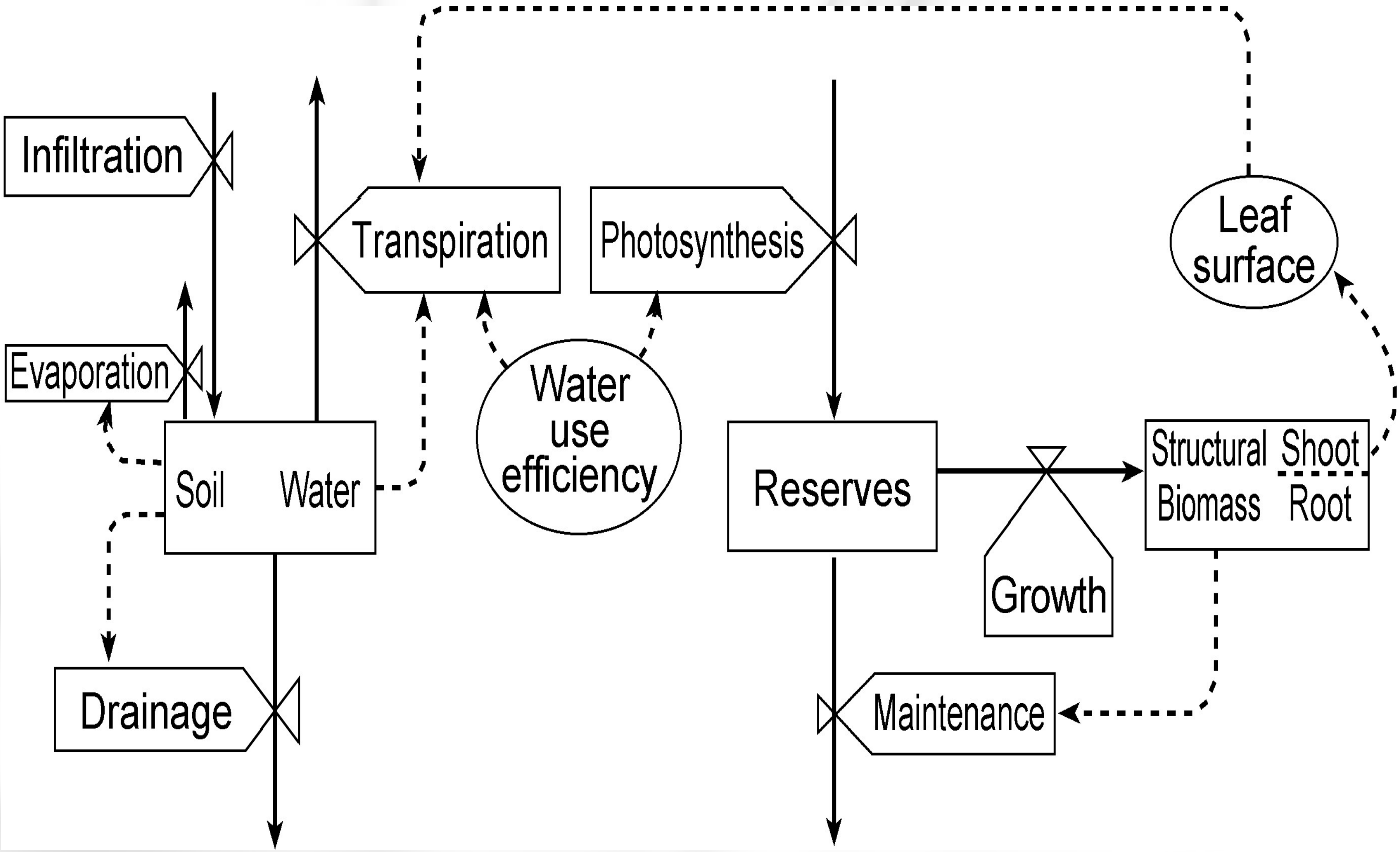
Timing Diagrams



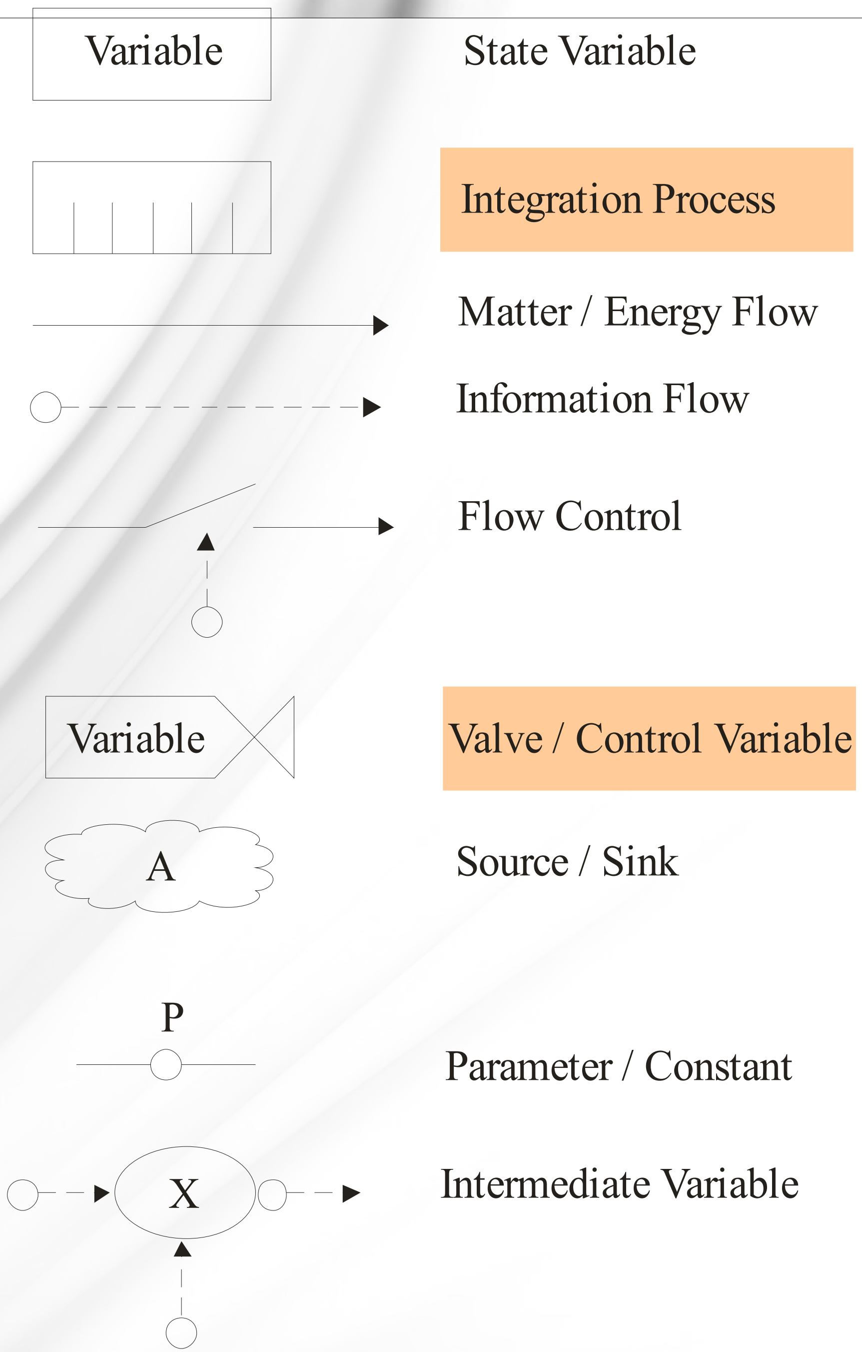
Continuous Dynamics



FORRESTER diagram



Forrester Symbols



Mathematical models

in

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Why do WE should use MATH ?

..to understand System Dynamics !

Key concepts
Derivative & Integrals

DERIVATIVE : dx(t) /dt

..is a ratio with the meaning of a rate, like a velocity
.. accumulation, assimilation RATE,
.. allows to observe how x(t) CHANGES

INTEGRAL : ∫ x(t) dt

.. almost a summation, an accumulation, like growth
..give the results of x(t) at the end of its life

The Power of ANALOGIES

ANALOGY stands for likelihood, similarity in structural / functional way

Using ANALOGY is describing a NEW system structure and behaviour by means of a WELL KNOWN one

..parable ..allegory .. fable .. tale


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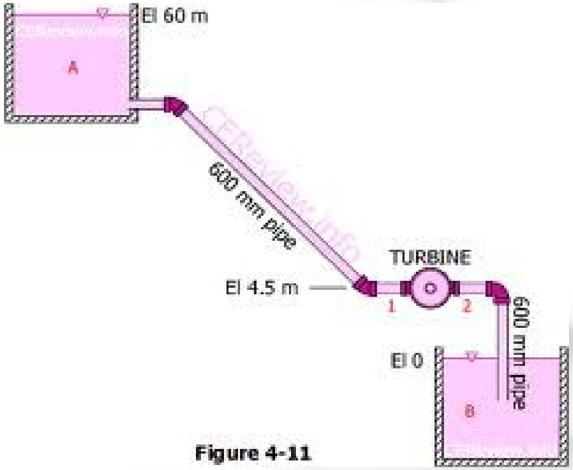
The GRASSHOPPER
AND THE ANTS

SIMPLE systems from hydraulics and thermodynamics

Flow vs pressure , vs level



$Q = K_p P$

$Q = K_h \, dh/dt$

$H = H_i + Q/k_h \, t$

Heat vs Temperature

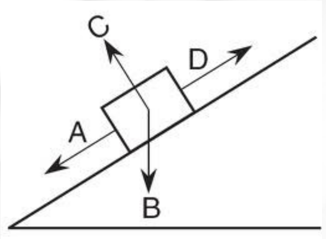
$Q = K_T \, dT/dt$

$T = T_i + Q/K_T \, t$

Simple systems:
.. from solid body mechanics

$F(t) = m\, a(t)$

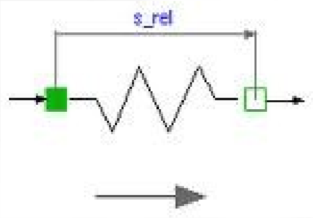
inertia



$F(t) = b\, v(t)$

friction

$F(t) = k\, x(t)$



elasticity

The exponential growth

- no limit to available resources
- net growth rate g=b-d
- no environmental influences

$$dy/dt = b\,y - d\,y$$

$$dy/dt = g\,y \quad \leftarrow \quad g=b-d$$

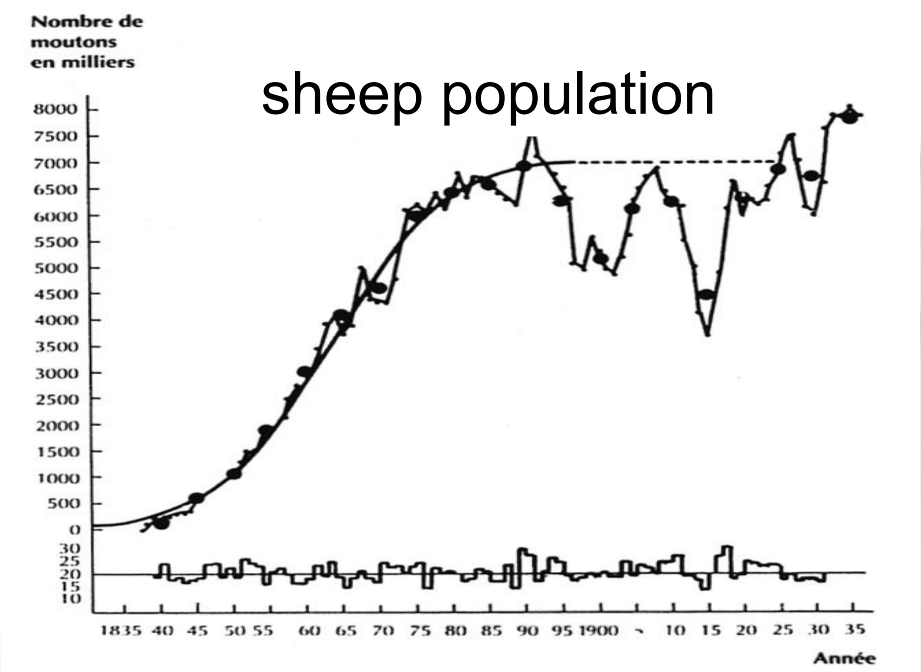
y: biomass
t: time
b: birth rate
d: death rate
g: net grow rate

.. has a solution

$$Y = A\,e^{gt}$$

The logistic growth

the size of population limits population growth



$$dy/dt = g\,y - p\,y^2$$

$$dy/dt = r\,y(1 - y/k)$$

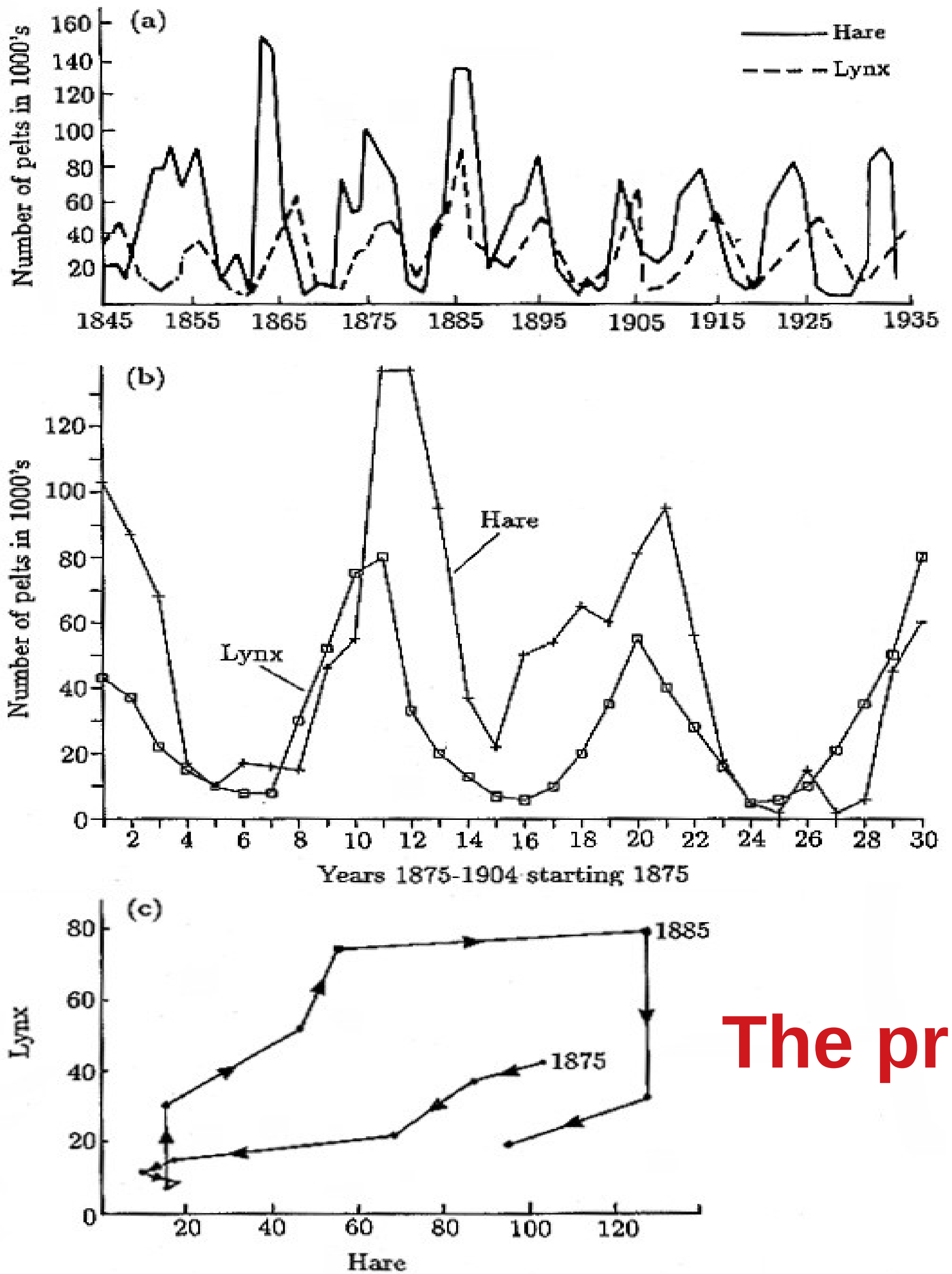
r: net grow rate

k: carring capacity

.. solution is:

$$Y = K / [\,1 + (K/Y_o - 1)\,e^{-r\,t}\,]$$

Hare-Lynx population size (Canada)

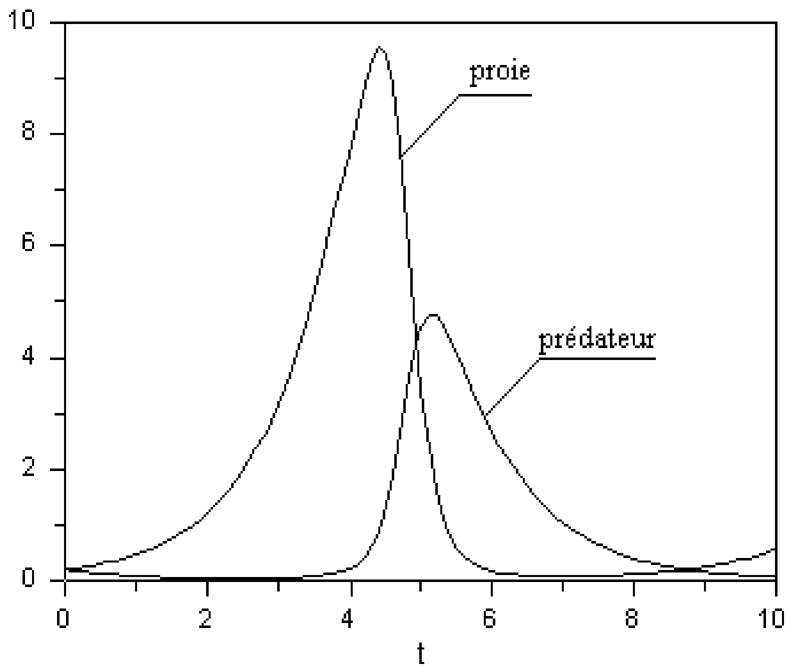


The prey predator system

Lotka-Volterra Model

$$\frac{dx}{dt} = r\,x - axy$$

$$\frac{dy}{dt} = -m\,y + b\,x\,y$$

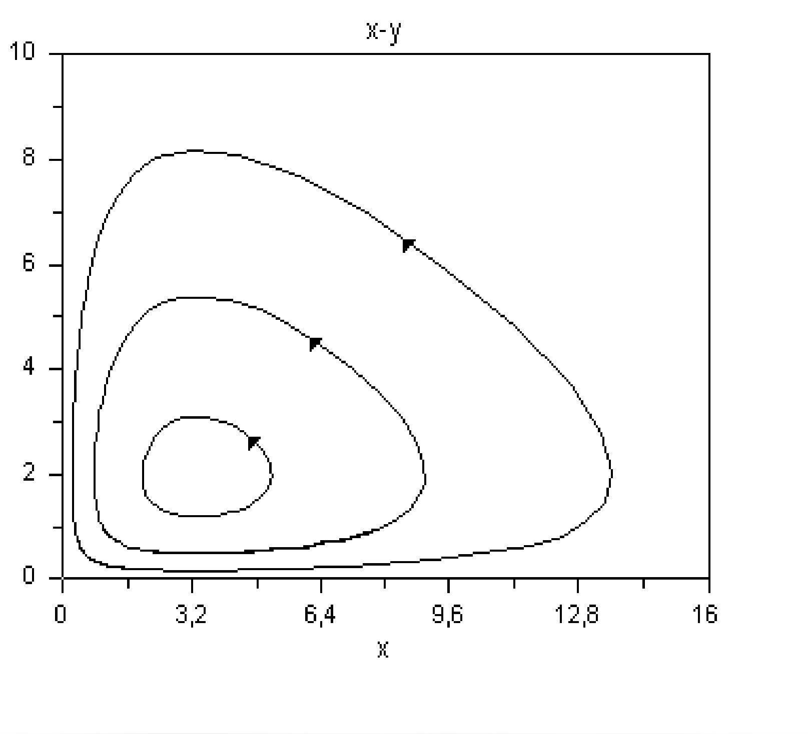


- x : prey
- y : predator
- r prey growth rate : Malthus law
- m predator mortality rate : natural mortality
- Mass action law
- a and b predation coefficients : b=ea
- e prey into predator biomass conversion coefficient

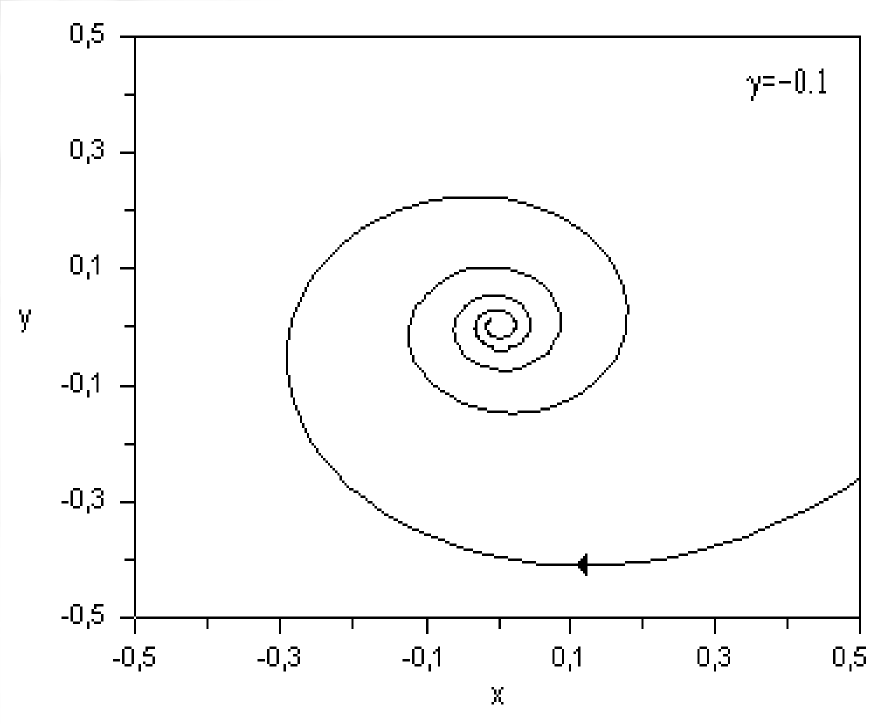
.. solved numerically

Solutions for Lotka-Volterra model

Limit Cycle



Attractor



..in this section we have seen

- model vs software/app
- modeling languages
- mathematical models
  - flow& rate
  - derivative & integrals
- analogies
- system dynamics
  - exponential growth
  - logistic growth
  - prey-predator system